Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix by

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$
 by applying elementary row transformations.

(06 Marks)

b. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Caylery-Hamilton theorem.

(05 Marks)

c. Solve the following system of equations by Gauss elimination method.

$$2x + y + 4z = 12$$
, $4x + 11 - z = 33$,

$$8x - 3y + 2z = 20$$

(05 Marks)

2 a. Find the rank of the matrix

$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

by reducing it to echelon form.

(06 Marks)

b. Find the eigen values of $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \end{bmatrix}$

(05 Marks)

c. Solve by Gauss elimination method: x + y + z = 9

$$x - 2y + 3z = 8$$
,

2x + y - z = 3

(05 Marks)

Module-2

3 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$

(05 Marks)

b. Solve $y'' - 4y' + 13y = \cos 2x$

(05 Marks)

c. Solve by the method of undetermined coefficients $y'' + 3y' + 2y = 12x^2$

(06 Marks)

OR

4 a. Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$

(05 Marks)

b. Solve $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$

(05 Marks)

c Solve by the method of variation of parameter $\frac{d^2y}{dx^2} + y = \tan x$

(06 Marks)

Module-3

5 a. Find the Laplace transform of

i)
$$e^{-2t} \sin h \, 4t$$
 ii) $e^{-2t} (2 \cos 5t - \sin 5t)$

(06 Marks)

b. Find the Laplace transform of $f(t) = t^2$ 0 < t < 2 and f(t + 2) = f(t) for t > 2.

(05 Marks)

c. Express $f(t) = \begin{cases} t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$ interms of unit step function and hence find L[f(t)]. (05 Marks)

OR

- 6 a. Find the Laplace transform of i) t cosat ii) $\frac{\cos at \cos bt}{f}$ (06 Marks)
 - b. Given $f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}$ where f(t+a) = f(t). Show that $L[f(t)] = \frac{E}{S} \tanh\left(\frac{as}{4}\right)$. (05 Marks)
 - c. Express $f(t) = \begin{cases} 1 & 0 < t < 1 \\ t & 1 < t \le 2 \end{cases}$ interms of unit step function and hence find L[f(t)].

(05 Marks)

Module-4

- 7 a. Find the inverse Laplace transform of i) $\frac{2s-1}{s^2+4s+29}$ ii) $\frac{s+2}{s^2+36} + \frac{4s-1}{s^2+25}$ (06 Marks)
 - 5. Find the inverse Laplace transform of $\log \sqrt{\frac{s^2 + 1}{s^2 + 4}}$ (05 Marks)
 - c. Solve by using Laplace transforms $y'' + 4y + 4y = e^{-t}$, given that y(0) = 0, y'(0) = 0.

 (05 Marks)

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- 8 a. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)(s+3)}$. (06 Marks)
 - b. Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+a}{b}\right)$. (05 Marks)
 - c. Using Laplace transforms solve the differential equation y''' + 2y'' y' 2y = 0 given y(0) = y'(0) = 0 and y''(0) = 6. (05 Marks)

Module-5

- a. State and prove Baye's theorem.
 b. The machines A, B and C produce respectively 60%, 30%, 10% of the total number of items
 - of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine "C". (05 Marks)
 - c. The probability that a team wins a match is 3/5. If this team play 3 matches in a tournament, what is the probability that i) win all the matches ii) lose all the matches. (05 Marks)

OR

- 10 a. If A and B are any two events of S, which are not mutually exclusive then $P(A \cup B) = P(A) + P(B) P(A \cap B)$. (06 Marks)
 - b. If A and B are events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$, $P(\overline{A}) = 5/8$. Find P(A), P(B) and $P(A \cap \overline{B})$.
 - c. The probability that a person A solves the problem is 1/3, that of B is 1/2 and that of C is 3/5. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved? (05 Marks)